Red Shift in Light

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ABSTRACT

The phenomenon of redshift of light is taught in undergraduate classes. The redshift of light occurs in three different contexts called Doppler redshift, cosmological redshift and gravitational redshift. In each case, the dynamics is distinct and involves different physical concepts. These must be discussed together so that one gets a comprehensive understanding of the subject. In this article, we describe each of these phenomena in a pedagogical manner without any advanced mathematics.

The phenomenon of ‘red shift’ more often alludes to the Doppler Effect in light. Doppler Effect is popularly known in context of sound waves. The pitch (or frequency) of the siren of a train (or a car) is higher when it is approaching a listener standing on ground and suddenly changes to a lower value as the engine of train (or the car) passes by the listener and moves away. This change in frequency of sound due to relative motion of source towards or away from the observer is known as Doppler Effect. Similar effect is observed in light also. The observed frequency of light, emitted by a source, changes due to relative motion between the source and the observer. However, as we shall discuss in this article, the phenomenon of red shift in light is very rich; besides Doppler red shift there also
occur the phenomena of cosmological and gravitational red shifts. The descriptions of all these phenomena lead to several other interesting concepts.

1. Doppler Redshift

In case of Doppler Effect in sound waves, if medium (say air) is still, the source is at rest, and observer is moving away from the source with constant speed \( v \) along line joining the source and the observer, then the observed apparent frequency \( \nu \) by the observer is given by

\[
\nu = \left( \frac{u - v}{u} \right) \nu_0 \tag{1}
\]

where, \( \nu_0 \) is the frequency of sound if the source is at rest with respect to the observer and \( u \) is speed of sound in still medium. On other hand, if the observer is at rest and the sound source is moving away from the observer, then observed frequency is:

\[
\nu = \left( \frac{u}{u - v} \right) \nu_0 \tag{2}
\]

Thus, while in both of the above cases, the relative velocity \( v \) between the source and the observer is same, the observed frequencies differ depending upon whether the source is moving or the observer. This asymmetry, which is apparently against the principle of relativity, arises because sound waves travel in and with respect to a material medium.\(^1\,^2\) Asymmetry lies in the fact that in one case, source is at rest with respect to medium while in the other case, it moves relative to medium. The two cases are therefore analyzed differently: when the observer moves away from the source, it receives lesser number of wave pulses per second than emitted by the source thereby effectively observing less frequency, while when the source moves away from the observer, undulations in medium get stretched thereby effectively increasing wavelength. In contrast, Doppler Effect in light, which requires no medium for propagation, is same in both the cases.

Doppler Effect in light essentially requires a relativistic analysis as described below; after all when light source is in motion, one should better do special relativity. Though relativistic Doppler Effect is discussed in the texts dealing with special relativity\(^2,^3\) it is instructive to recapture its basic steps. Suppose frames \( S \) and \( S' \) are attached to observer and source respectively. Let \( S' \) be moving away from \( S \) (rest frame) with constant velocity \( v \), along common \( X-X' \) axis. The source emits monochromatic, harmonic waves of frequency \( \nu_0 \) in frame \( S' \). The first concept that goes into the analysis is that we can regard the light source as a clock which ticks regularly at time interval \( \tau_0 = 1/\nu_0 \), emitting a harmonic pulse (sine wave) of light at each tick. If space-time coordinates of emission of two consecutive pulses in \( S' \) frames are \((x_1', t_1')\) and \((x_2', t_2')\), then \( x_2' = x_1' \) and \( t_2' = t_1' + \tau_0 \). For the observer \( S \) these events occur at coordinates \((x_1, t_1)\) and \((x_2, t_2)\). Lorentz transformations between \( S \) and \( S' \) yield

\[
\Delta x = x_2 - x_1 = \gamma v \tau_0 \\
\Delta t = t_2 - t_1 = \gamma \tau_0
\]

where \( \gamma = \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \) is the Lorentz factor. The observer in frame \( S \) finds that two light pulses reach him separated by time interval \( \tau \) such that

\[
\tau = \Delta t + \Delta x/c = \gamma (1 + v/c) \tau_0 \tag{3}
\]

The time \( \tau \) contains two terms: \( \Delta t \) because two pulses are emitted at different times and \( (\Delta x/c) \) because second pulse has to travel an extra distance \( \Delta x \) in order to reach the observer. The observed frequency by observer \( S \) at rest therefore is

\[
\nu = 1/\tau = \left[ \frac{1}{\gamma (1 + v/c)} \right] \nu_0 \\
= \left[ \frac{c - v}{c + v} \right]^{1/2} \nu_0 \tag{4}
\]
As beautifully shown by R.P. Feynman, the effect is same if we take source at rest and observer moving; the $\gamma$ factor plays the trick.

However in the non-relativistic limit ($v<<c$, neglecting terms of $(v/c)^2$ and higher powers) of the relativistic Doppler Effect in light, the observed apparent frequency is given by same relation as Eq.1(or 2) as expected, with speed of sound $u$ replaced by that of light $c$:

\[ \Delta \nu = \nu_0 - \nu = (v/c) \nu_0 \] (5)

Corresponding change in wavelength is

\[ \Delta \lambda = \lambda - \lambda_0 = (v/c) \lambda_0 \] (6)

Thus the observed wavelength increases if the light source is moving away from the observer. Since the wavelength of red light is larger in the visible band, this phenomenon is called ‘red-shift’, i.e. shift towards larger wavelength.

One of the most important examples of Doppler redshift is the Doppler broadening of emission lines from excited atomic (or molecular) gas. At temperature $T$, different molecules of the gas move randomly in all directions with different speeds. Any emission line therefore is Doppler shifted because the source is moving. The shift depends upon the speed of a molecule. The net spectral profile turns out to be Gaussian peaked at frequency $\nu_0$ and its full width at half maximum is given as

\[ \Delta \nu = \nu_0 (2kT \ln 2/Mc^2)^{1/2} \]

where $\nu_0$ is the frequency if molecules (of mass $M$) were at rest.

2. Cosmological Redshift

The second exciting example of redshift in light is seen in the spectra of the radiation emitted by distant galaxies. In the beginning of the last century it was observed by the astronomers that the light emitted by the distant galaxies is red-shifted. This discovery has led us to infer that our universe is expanding in which all the galaxies are moving away from each other.

It is reasonable to believe that any typical galaxy shines because of the radiation emitted by billions of stars in it and that all typical stars anywhere have similar composition as stars in our own galaxy. Therefore spectrum of radiation from any other galaxy is expected to exhibit same pattern of emission lines as seen in the spectra of our nearby stars or galaxies. The most strong lines usually found in these systems are those belonging to ionized H, He, Ca, K, etc. We do find the patterns corresponding to these lines in spectra of any distant galaxy (situated in any direction) but exact values of wavelengths of these lines turn out to be larger by a fixed ratio (for a given galaxy) compared to the wavelengths of respective lines observed in the laboratory. That is, for every observed line, from a particular distant galaxy, one finds that $\lambda/\lambda_0 = 1 + z$, where $z$ is called the redshift parameter.

According to Doppler analysis, the value of $z$ is related to the value of the speed with which that galaxy is moving away from us as given by Eq.5. This led to the conclusion that our universe is expanding.

However the interpretation of redshift of galaxies as Doppler shift implies that we are using special relativity, that is, a single Lorentz space-time coordinate system (with ourselves at the origin) all the way up to source galaxy. This interpretation is approximately valid for nearby galaxies. For distant galaxies, one must take into account space-time curvature and use general relativity to explain galactic red shifts. The general relativity model argues that galaxies are not moving away from us ‘in space’. Rather galaxies appear to be receding from us because the space itself is expanding as a consequence of Big Bang origin of space-time. This aspect is popularly visualized in terms of inflation of the (two dimensional) surface of a balloon, pock marked with dots.
representing various galaxies. The balloon example illustrates two points: first that expansion causes any two dots on its surface to recede from each other as the balloon inflates and secondly that the expanding surface need not have any center and edge. Similarly, our three-dimensional space is expanding and has no center and edge. It is a consequence of the Cosmological Principle.

The Cosmological Principle asserts that universe on the whole is both homogeneous and isotropic as seen from any point (within it) and hence all the galaxies are receding not just away from us (as we do not constitute any special center of the universe) but they are all receding from each other. The shift is indeed a ‘cosmological redshift’. As the space expands or stretches, the length of a wave stretches. The wavelength changes in proportion to the scale of the length dimension of the space; that is, we have

\[ \frac{\lambda(t_2)}{\lambda(t_1)} = \frac{S(t_2)}{S(t_1)} \]

\[ = 1 + z \quad (7) \]

\( \lambda(t_1) \) is the wavelength emitted by source galaxy at (cosmic) time \( t_1 \) and \( \lambda(t_2) \) is the wavelength observed by us at time \( t_2 \) (\( t_2 > t_1 \)); \( S(t) \) is the scale factor that represents measure of length in space which changes with time as space expands. For example, between times \( t_1 \) and \( t_2 \), if the universe becomes ten times in size, the observed wavelength too increases ten times, i.e. \( z=9 \). This can’t be interpreted simply as a source moving with speed 9 times that of light (at cosmic time \( t_2 \) (or \( t_1 \)). Cosmological redshift provides information only about change in scale-factor. Cosmic redshift \( z=9 \) means that universe expanded by 900% between emission and reception of light signal; the signal traveled during all this time in an expanding space. It is indeed a phenomenon emerging from general relativity, which of course is the right thing to do if we are talking about the motion of cosmos.

Another important example of cosmic redshift is the cosmic background microwave radiation (CMBR) observed today with peak intensity at wavelength of about \( 2 \times 10^{-3} \)m. CMBR corresponds to a redshift of \( z \sim 1000 \). It simply means that universe has expanded 1000 times since primeval radiation (with peak at wavelength\(-2 \times 10^{-6} \)m) from hot big bang decoupled itself from matter and this radiation has been moving along with expanding space, getting stretched in the process.

The Doppler shift given by Eq.5 is the result from special theory of relativity in which case the speed of receding galaxy cannot exceed the speed of light \((v < c)\) since \( v \) represents the speed of the source as it travels through ‘in space’. In case of cosmological redshift, the speed with which a galaxy is receding may be more than \( c \) because it represents the speed with which the space is expanding. However there is a word of caution here: the cosmic redshift parameter \( z \) is not equal to \( v/c \). In fact there is no unique relation between \( z \) and speed of receding galaxy at the time of emission of light signal; Eq.7 says nothing about the speeds of source and observing galaxies at times \( t_1 \) and \( t_2 \). The speed of any receding galaxy depends on how expansion occurs, that is, on the model of universe we use for study of expansion.

3. Gravitational Red Shift

Another, perhaps more exciting example, of redshift in light is the shift caused by motion of light in a gravitational field. When a beam of light moves, say upwards from the surface of earth against the direction of its (earth’s) gravitational field, its wavelength increases. This phenomenon is called gravitational redshift.

The understanding of gravitational redshift has several interesting ideas associated with it at different levels of sophistication. The simplest situation, as mentioned above, is to consider a (weak) uniform gravitational field
(say of earth) in which two observers P and Q are sitting at rest with respect to each other, P on surface of earth and Q at a vertical height \( h \) above P. A light source at P emits light waves of frequency \( \nu_p \). Observer Q finds that frequency of these waves is \( \nu_Q \) which is less than \( \nu_p \) that is, light gets red shifted as seen by observer Q. A simple text-book explanation of this phenomenon is given by invoking Equivalence principle\(^{4,5} \) which says that uniform gravity is equivalent to a non-inertial frame falling freely with acceleration ‘\( g \)’ in a gravity free space. Thus we consider another observer R who is initially sitting at rest along with Q. The moment P emits a wave, R falls freely. For R, it is gravity free space, and hence he observes that the wave is moving with constant frequency \( \nu_P \) all the way from P to Q. However, as R falls down, he notes that Q is moving up, away from the light source and therefore R concludes that the light observed by Q must be Doppler shifted and is given by Eq. 3:

\[
\nu_Q = \nu_P \left(1 - \frac{v}{c}\right) = \nu_P \left(1 - \frac{gh}{c^2}\right) 
\]  

(8)

where \( v = gt = gh/c \) is the speed of Q when wave reaches him (Q). The above analysis, done within the framework of non-relativistic Newtonian gravity, is a way to interpret gravitational redshift in terms of motion of the observer (i.e. as Doppler shift). But the subject has deeper significance.

We have discussed above that a light source is essentially a clock which ticks with time-period \( \tau = 1/\nu \). Suppose, therefore, that we have two identical clocks out of which one is given to P and the second is taken to Q at height \( h \). Now, clock P ticks N times in time \( t_P = \tau_P N \). Observer Q receives these \( N \) waves in time \( t_Q = \tau_Q N \). If \( \nu_Q < \nu_P \), we get \( \tau_Q > \tau_P \) and hence \( t_Q > t_P \). This implies that during the process of emission and receiving of \( N \) light waves by P and Q respectively, the clock of Q has recorded more time. That is, the clock Q runs faster as compared to clock P. Different clocks (otherwise identical), relatively at rest, run at different rates depending upon where they are located in gravity. The above analysis indicates that clock P runs slower since it lies in the region of higher gravity, being closer to source of gravity. The big result is that gravity slows down the flow of time; it is time dilation in gravitational field.

Eq.8 was obtained in context to uniform gravity. We can extend it to gravity of a uniform spherical source of mass \( M \). Consider an observer Q situated very far from it, in gravity free space, defining a space-time coordinate system \( x-t \). The coordinate time \( t \) is the proper time recorded by his clock. Consider another observer P situated at distance \( r \) from the source so that \( g = (GM/r^2) \), the time recorded by the clock at P is its proper time \( \tau \). Eq.8 suggests that the clock of P runs slower (being in gravity) such that

\[
\Delta \tau = \Delta t \left(1 - \frac{gr}{c^2}\right) = \Delta t \left(1 - \frac{GM}{rc^2}\right) 
\]  

(9)

Note that \( \Delta \tau = \Delta t \) when \( r \to \infty \). We define \( \Delta t \) as the universal coordinate time recorded by a clock situated very far \( (r \to \infty) \) from all the sources, in gravity free space. The (proper) time recorded by a clock sitting at distance \( r \) from a source of gravity (say a star) is \( \Delta \tau \) which is less than \( \Delta t \). The clock in gravity runs slower and the time dilation factor is \( (1 - GM/rc^2) \).

In the language which emerged after Einstein articulated that gravity is space-time curvature, the gravitational redshift is the manifestation of curvature of time. Time flows differently at different positions means that time is curved. Since space-time interval squared is negative of proper time squared, we find that for any arbitrary observer at rest (spatial coordinates being fixed) in (weak) gravity.
Δr^2 = -(1 - GM/rc^2)^2 Δt^2
≈ -(1 - 2GM/rc^2)Δt^2 (10)

Note that above analysis, done within Newtonian framework, shows how Newtonian (i.e. weak) gravity affects the flow of time. However it turns out that general relativity applied to strong gravity sources also yield same result (Eq.10).

As mentioned above, curvature of time (revealed as gravitational red shift), is produced even by such weak sources as earth. It was verified in laboratory by Robert Pound and Glen Rebka in 1960 in the famous experiment using Mossbauer resonance spectroscopy. They observed the frequency shift of gamma rays emitted by an isotope of iron (embedded in a cool crystal) at a vertical separation of 22.5m. Variation of clock rate with position in earth’s gravity is also taken into account in defining global positioning system used by satellites. Gravity also curves space but that is outside the subject of present article.

Before we close this discussion, we must mention that Eq 8 is often described it terms of energy conservation applied to photon. As a photon moves up against the gravity, its energy hν_0 decreases to hν. What happens to its energy loss? This is explained in a simple way by saying that it is converted into gravitational potential energy of the photon. The total (dynamic) energy of the photon is defined as the sum of hν and the gravitational potential energy GMm/r of the photon (whatever it means), where m= hν/c^2 denotes effective gravitational mass of the photon. Hence energy conservation implies that

hν_0 = hν + GMm/r = hν(1 + GM/rc^2)

which leads to Eq.9 up to first order. This, as S. Weinberg writes, is a way to put quantum theory, energy conservation, and Newtonian gravity together. In relativistic theory of gravitation (general relativity), radiation energy density is an intrinsic source of gravity.

References